

XIII. *Remarks on specific Gravities taken at different Degrees of Heat, and an easy Method of reducing them to a common Standard.* By Richard Kirwan, Esq. F. R. S.

Read February 17, 1785.

**T**HAT a comparative view of the weights of equal volumes of water and all other substances is highly useful on many occasions, is too well known to require any proof; but that a principal use resulting from this comparison, when properly made, is unattainable by a perusal of the common tables, I shall here endeavour to shew, and at the same time point out a remedy for this defect.

One capital advantage derivable from a table of specific gravities, is the knowledge of the absolute weight of any solid measure of the substances therein contained, or that of the solid measure of a given weight of those substances, a cubic foot of water being supposed to weigh 1000 ounces avoirdupois, and consequently a cubic inch of water weighing 253,182 grains. But the authors who have discovered this equation of weight and measure, and all those who have since treated this subject, have neglected to inform us of the temperature at which this agreement takes place; yet that it cannot take place in all temperatures is evident from the experiments of Dr. HALLEY and others, who have found, that from a few degrees above the freezing to the boiling point, water is dilated about  $\frac{1}{7}$  of its bulk; and, consequently, if 1000 ounces at the freezing point be equal to one cubic foot, they must be equal at the boiling

point to one cubic foot and 66,46 cubic inches. And if the dilatations are proportional to the degrees of heat throughout the scale, there must be an augmentation of 3,136 cubic inches *per* cubic foot, produced by every 10 degrees of heat. Both these points remain, therefore, to be determined; first, at what temperature a cubic foot of water weighs exactly 1000 ounces avoirdupois; and, 2dly, whether the dilatations produced by successive degrees of heat are proportional to the degrees that produce them. This last point has indeed been handled by others, but with different views; and their determinations are not easily applicable to the present question.

To examine this matter experimentally, I ordered a hollow tinned iron cone to be made, of four inches diameter in the base, one-tenth of an inch diameter in the summit inside, and 10 inches perpendicular height, whose solid contents should be 42,961 cubic inches, but by a slight diminution of the diameter, and a protuberance arising from the soldering, I found it to contain, in the temperature of  $62^{\circ}$ , but 42,731 cubic inches, according to the estimation of 1000 ounces to the cubic foot; and having filled it by immersion in boiling water, and taken it up at different degrees of heat, and weighed it when cold, I found its contents as expressed in the following table; the first column of which shews the degrees of heat at which it was taken up; the second, the weight of the water contained in it; the third, the diminution of weight occasioned by those degrees of heat; the fourth, the sum of the diminutions of weight in the cubic foot, by the preceding degrees of heat; the fifth shews the weight of a cubic inch of water in each of those degrees of heat; and the sixth, the augmentation of bulk in the cubic foot by every  $20^{\circ}$  of heat. The horizontal lines, marked thus \*, I have added from the experiments of Mr.

BLADH, in the Memoirs of the Academy of Stockholm for the year 1776, whose determinations, as far as they reached, agreed very nearly with mine. The water I used was common water well boiled and filtered. The experiments were for the most part three times repeated, and the difference in each trial amounted to a very few grains.

I.	II.	III.	IV.	V.	VI.	
Degrees	Contents of the cone in grains.	Dimin. in grains.	Sum of dim. in a cubic foot.	Weight of a cubic inch.	Increase in cubic inches.	
212	10418,75	29,5	16589	243,8	4,892	
202	10448,25	77,5	15354	244,51	12,818	
182	10525,75	71,75	12133	246,33	11,533	
162	10596,00	62,60	9171	247,97	10,209	
142	10658,60	56,15	6602	249,43	9,103	
122	10714,75	49,00	4310	250,75	7,920	
102	10763,75	35,5	2226	251,89	5,7	
82	10799,25	19,5	788	252,72	3,120	
*75	— — —	— — —	— — —	252,8	— — —	
*70	— — —	— — —	— — —	252,97	— — —	
*66	— — —	— — —	— — —	253,06	— — —	
62	10818,75	0	0	253,182	0	Total increase of bulk from 62° to 212° = 65,327 cubic inches.
*56	— — —	— — —	— — —	253,3	— — —	
		Increase	Increase			
*50	— — —	— — —	— — —	253,46	— — —	Decrease Total from 36° to
42	10830,75	12	485,3	253,463	1,936	212 = 67,327 cubic inches.
*36,5	— — —	— — —	— — —	253,5	0,064	

Hence we see, that a cubic foot of water weighs 485,3 grains more at 42° than at 62°, and consequently is equal to 1001,109 avoirdupois ounces, and in the temperature of 82° it weighs less than at 62° by 788,5 grains, and therefore is equal to 998,198 ounces. At the boiling point it wants 16589 grains, or 37,915 ounces of the weight it possesses at 62°, and consequently weighs but 962,085 ounces, &c.

In this calculation I take no account of the difference arising from the expansion of the vessel, it being only 0,067 of an inch at the boiling point; for, according to BOUGUER, iron is dilated 0,00055 of its bulk from the freezing to the boiling point; consequently 42,961 cubic inches gain only 0,067 of an inch, augmenting the diameter and perpendicular height of this frustum of a cone at the boiling point in that proportion.

Hence also we see, that the expansions of water are not proportional to the degrees of heat; for by 20 degrees of heat from 62° to 82° a cubic foot of water is dilated only 3,12 inches, but by the next 20 degrees of heat, that is, from 82° to 102°, it is expanded 5,7 inches, &c.

Mr. BLADH found the volume of water at 32° to be equal to that at 53°, 6; but that this irregular expansion ceased at 36°, 6, and, according to Mr. DE LUC (who first discovered it) at 43°.

As the expansion of liquids by equal degrees of heat is much greater than that of solids, it happens, that the specific gravities of the same solid taken at different temperatures will be different; and, what appears more extraordinary, the same solid will appear specifically heavier in higher than in lower temperatures; for the same volume of water being lighter in higher than in lower temperatures, the solid will lose less of its weight in it in the former than in the latter case: this mistake we may remedy by inspecting the fifth column of the foregoing table and the following analogy: as the weight of a cubic inch of water at the temperature of 62° is to the weight of a cubic inch of water at  $n$  degrees of temperature, so is the specific gravity found at  $n$  degrees of temperature to that which will be found at 62°.

Thus, if 1000 grains of iron be weighed in water of the temperature of 62°, and it loses therein 13,333 grains, if the same

same piece of iron be weighed in water of the temperature of  $75^{\circ}$ , it will lose but 13,313 grains; for the losses of weight will be as the weights of equal volumes of water at those temperatures, which, as we have seen, are as 253,18 to 252,8; therefore, its specific gravity in water of the temperature of  $62^{\circ}$  will be 7,49; and in water of the temperature of  $75^{\circ}$ . 7,511; but we may correct this by the above analogy, for  
 $\therefore 253,8 : 252,18 :: 7,511 : 7,49.$

By this means we obtain the advantage of discovering the true weight of a cubic foot of any substance whose specific gravity is known, which it is now plain cannot be known when bodies are hydrostatically weighed at any temperature a few degrees above or below  $62^{\circ}$ , without such reduction, or subtracting the quantities in the fourth column.

This method is equally applicable, and with equal necessity, to other means of finding specific gravities, as areometers, the comparison of the weights of equal measures of liquids, the different losses of weight of the same solid, when weighed in different liquids, &c. In all which cases the weight of water at  $62^{\circ}$ , or the loss of weight of a solid in water at  $62^{\circ}$ , should be found by the above analogy.

Dr. HALES and some others have estimated the weight of a cubic inch of water at 254 grains, which is an evident mistake, as it is true in no degree of temperature, and produces an error of more than three ounces in the cubic foot.

